EFFECTIVENESS OF PROBLEM POSING STRATEGIES ON PROSPECTIVE MATHEMATICS TEACHERS' PROBLEM SOLVING PERFORMANCE

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Problem solving has long been viewed as an important topic in mathematics education. It is a focus issue concerning students' learning of mathematics (NCTM, 1989, 1991). Contemporary reform efforts not only place a heavy emphasis on problem solving but also on problem solving but also on problem posing. In the Curriculum and Evaluation Standards, problem solving is included as the first in the list of standards across all grade levels. In particular, it suggested the "investigating and formulating questions from problem situations" by students themselves. (NCTM, 1989, 70). In the Professional Teaching standards, it proposed that "students should be given opportunity to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (NCTM, 1991, 1995). The suggestions in both standards imply that problem posing is an integral part of problem solving and should not be emphasized separately from problem solving. The purpose of this study was to examine the effectiveness of carrying out problem posing strategies on prospective mathematics teachers' problem solving performances and, especially to find out whether there were differences between student teachers who used problem posing strategies and those who did not. The results of this study showed the performance of student teachers improved overall when using problem posing strategies.

INTRODUCTION

One of the major goals in mathematics teaching is to encourage our students to be good problem solvers. To achieve that goal teachers have to teach mathematical problem solving strategies with more practice. Mathematics educators tend to neglect the other side of the coin in mathematical problem solving in mathematics teaching programmes, that is problem posing

(Gonzales, 1994), in spite of its importance in developing our students' mathematical thinking. New trends in mathematics education (NCTM, 2000) recommend a change from asking students to solve problems, to developing problems through changing their questions, adding new data, eliminating some data, changing variables or constructing a new problem based on the original idea.

In the author's discussions with teachers, the author observed that their abilities in solving non-routine problems were very weak. But they had a positive attitude to pose questions from a given problem. The author tried to give more attention in mathematical problem solving when posing a topic in "Methods of Teaching Mathematics" for prospective teachers in the College of Education.

OBJECTIVES OF THE STUDY

1) To identify the effectiveness of using problem-posing strategies on performance of prospective mathematics teachers for problem solving.
2) To identify problem posing skills needed to be included with Polya's four steps to improve mathematics for prospective teachers in problem solving performance.
3) To develop educational activities for mathematical problem solving and posing as a part of a mathematics education programme for prospective teachers.

BACKGROUND

The first recommendation in "An Agenda for Action" produced by the NCTM in the US, recommended problem solving be the focus of school mathematics in the 1980s. School Mathematics should contain problem solving as the main activities in all mathematics aspects; also teachers should offer their students rich problems, often based in the real world, which would challenge and excite them, because problem solving is an effective way to introduce and explore new areas in mathematics. Through problem solving, the students can develop much of the mathematics for themselves.

Student teachers are prepared to teach mathematics with a problem solving approach, to help their students in solving mathematical problems. Their educational programme to do that doesn't reflect their abilities to solve problems. Abilities to use different problem posing strategies, may affect their problem solving performance.

Relationships between problem solving performance and problem posing still need to be explored as Silver and Cai (1993) mentioned "there is a need for further research that examines the complex relationship between problem posing and problem solving." There is also interest in exploring the relationship of posing to other aspects of mathematical knowing and mathematical performance.

In Silver's (1994) researches, he found different results of that relationship. Silver and Cai (1993) found a strong positive relationship between posing and solving performance. While Silver and Mamona (1989) found no overt link between the problem posing of middle school mathematics teachers and their problem solving abilities there is no clear, simple link established between competence in posing and solving problems (Silver, 1994). It is possible to improve student teachers' performance in problem solving, by using problem posing strategies. Kilpatrick discussed that and suggested that by drawing students' attention to the reformulating process and given practices in it, the students can improve problem solving performance (1987).

Given a mathematical problem to a student, means the student is put in a new thinking situation; thinking of the given information in the problem statement, thinking of a best strategy to solve it using his own questions that lead him to a solution and thinking of more information related to the given information.

The given information given explicitly in a problem statement is almost never adequate for solving the problem. The problem solver has to supply additional information consisting of premises about the problem context (Kilpatrick, 1987). For example, to solve a word problem about the distance between two cities, students need to understand that distance cannot be negative numbers.

The idea of improving mathematical problem solving performance has been discussed in the light of Polya's four steps for problem solving. Through problem posing in Polya's steps, problems can themselves be the source of new problems. The solver can intentionally change some or all of the problem conditions to see what new problems result, and after a problem has been solved the solver can "look back" to see how the solution might be affected by various modifications in the problem.

In "making a plan" to solve a problem, Kilpatrick (1987) showed that students may take Polya's heuristic to see whether, by modifying the conditions in the problem, a new, more accessible problem might result that could be used as a stepping stone to solve the original one.

Polya was looking towards problem solving as a major theme of doing mathematics, and "teaching students to think" was of primary importance. The other aspect of problem solving that is seldom included in textbooks is problem posing. Polya did not write specifically on problem posing, but much of the spirit and format of problem posing is included in his illustrations of "looking back" (Wilson, Fernandez & Hadaway, 1993). "Looking back" may be the most important part of problem solving. It is the set of activities that provides the primary opportunity for students to learn from the problem. Polya identified this phase with admonitions to examine the solution by such activities as checking the result, checking the argument, deriving the result differently, using the method for some other problem, reinterpreting the problem or stating a new problem to be solved.

Teacher's skills on using Polya's four steps in problem solving should be consistent with their abilities to use suitable problem posing strategies to generate more questions and problems for students.

MATHEMATICAL PROBLEM POSING STRATEGIES

Mathematics teachers might use one or more strategies to formulate new problems or encourage their students in mathematics classes to be good problem posers as well as a good problem solvers. Strategies could be used depending on the most suitable conditions (mathematics content, students' levels, learning outcomes and mathematical thinking types). Problem posing situations are classified as free, semi-structured or structured situations.

Free Problem Posing Situations

Situations from daily life (in or outside school) can help a student to generate some questions leading him/her to construct a problem. Students are asked to pose a problem to encourage them to "make up a simple or difficult problem" or "construct a problem suitable for a mathematics competition (or a test)" or "make up a problem you like." It is more useful if the teacher tries to relate the real life situations to the mathematics content being taught and to ask students to pose new problems. This will be more effective in developing students' mathematical thinking. Problem posing situations might take these types: every day life situation, free problem posing, problems they like, problems for a mathematics competition, problems written for a friend and problems generated for fun.

Semi-Structured Problem Posing Situations

Students are given an open-ended situation and are invited to explore it using knowledge, skills, concepts and relationships from their previous mathematical experiences and this can take the following forms:

Open-ended problems (i.e. mathematical investigation).

Problems similar to given problems.

Problems with similar situations.

Problems related to specific theorems.

Problems derived from given pictures.

Word problems.

This strategy was developed with student teachers as the following (Abu-Elwan, 1999):

1) A semi-structured situation from a student's daily life was presented to all students.
2) Students were asked to complete the situations using their perspective to be able to pose problems from that formed situation.

Students can generate problems by omitting the questions from given situations.

Structured Problem Posing Situation

Any mathematical problem consists of known (given) and unknown (required) data. The teacher can simply change the known and pose a new problem, or keep the data and change the required. Brown and Walter (1990, 1993) designed an instructional problem formulating approach based on the posing of new problems from already solved problems, but they have also recommended varying the conditions or goals of given problems.

This reformulation approach appears to be the most effective method for introducing structured problem posing activities in mathematics classrooms.

In order to create teaching/learning situations that provide a good problem posing situations, Lowrie (1999) recommended the mathematics teacher to:

1) encourage students to pose problems for friends who are at or near their own level until they become more competent in generating problems;
2) ensure that students work cooperatively in solving the problems so that the problem generator gains feedback on the appropriateness of the problems they have designed;
3) ask individuals to indicate the type of understanding and strategies the problem solver will need to use in order to solve the problem successfully before a friend generates a solution;
4) encourage problem solving teams to discuss, with one another, the extent to which they found problems to be difficult, confusing, motivating or challenging;
5) pProvide opportunities for less able students to work cooperatively with a peer who challenged the individual to engage in mathematics at a higher level than they were accustomed;
6) challenge students to move beyond traditional word problems by designing problems that are open ended and associated with real life experiences; and
7) encourage students to use technology (calculators, CDs, computers) in developing their mathematical thinking skills, so they can use this technology to generate new mathematical situations.

RESEARCH QUESTIONS

This study attempts to answer the following questions:

1) How effective is the teaching of problem posing strategies able to enhance student teachers' performance in problem solving?
2) Is there any differences in mathematical problem solving skills between student teachers who study problem posing strategies and those who just study problem solving strategies?

Hypotheses

The study included three hypotheses:

1) There is a statistically significant difference ($p<0.01$) between student-teachers' mean scores of the experimental group and the control group in the mathematical problem solving part of the achievement text in favour of the experimental group.
2) There is a statistically significant differences ($p<0.01$) between student-teachers' mean scores of the experimental group and the control group in mathematical problem posing part of the achievement test in favour of the experimental group.
3) There is a statistically significant differences ($p<0.01$) between student-teachers' mean scores of the experimental group and the control group in mathematical problem (solving-posing) test in favour of the experimental group.

METHOD

Subjects

Fifty student teachers participated in the study. All of the them were in Grade Three in the College of Education, (Sultan Qaboos University) and their major subjects were mathematics and computer science. They were enrolled in the "Methods of Teaching Mathematics 2" Course. They were divided in two groups. Group E as an experimental group and Group C as a control group: each group consists of 25 student teachers. The study was conducted over the period of time extending from November 12, 2000 to December 17, 2000.

Instruments

An achievement test on "Mathematical Problem Posing-Solving" had been developed to determine the effectiveness of using problem posing strategies to enhance student teachers' performance in mathematical problem solving. The main topics and ideas of the achievement test are relevant in the students' everyday life and are reinforced through common activities like providing shopping and vacationing problems developed based on the three problem solving strategies:

Look for a pattern,

Make a list,

Work backward.

The test consists of eight open-ended problems, each one of these problems contains:

A statement of the problem.

A question asking the students to solve it.

Another question asking students to pose an extension to the original problem, then to solve it.

Four referees were asked to give their opinions regarding the validity of the test. Based on their suggestions, all modifications were developed. The reliability of the test was established using a group of 20 mathematics student-teachers. A reliability coefficient mean value of 0.69 was secured.

The achievement test in its final form was then used in the experimental design.

The Experimental Design

First: Student-teachers in Groups E and C were involved in a quasi-experimental design as follows:

Student-teachers in Group C (control) studied a problem solving strategy based on Polya's four steps; several problems had been presented to student-teachers in this group, they were requested to solve the problems using suitable problem solving strategies. The problems had been chosen from:

- School mathematics textbooks (middle and secondary stages)
- Problem solving experiences in mathematics as a source book
- The Internet web: www3.actden.com/math-den

Second: Student-teachers in group E (experimental) studied problem solving strategies based on Polya's four steps (Polya, 1973). Problems had been chosen from the previous sources like group C. But students in group E, studied it under other treatment for Polya's four-steps.

Student-teachers explored the intent of each step in the process and attained a basic understanding of the process (Figure 1).

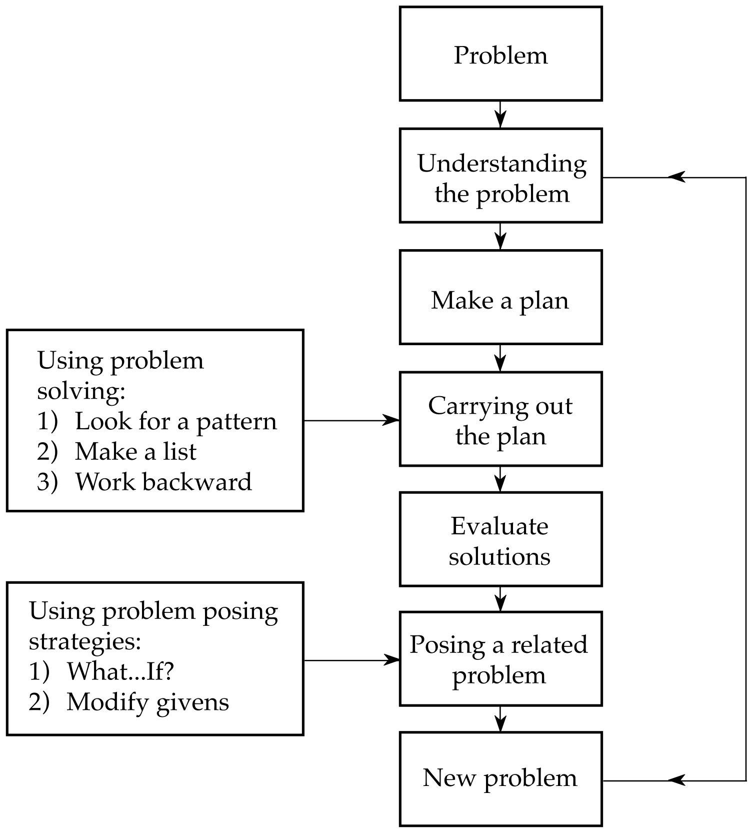

Figure 1: Framework for cycle of activities (solving-posing)

The students are introduced to solve the problems by traversing each of Polya's steps in the problem solving process.

They are required to write a description of each step as follows.

1) *Understanding the problem:*

   Ask yourself questions such as: "What is the problem all about?", "What am I given and not given?," "What do I need to find?"

2) *Make a plan:*

   What strategy of which you know (look for pattern, making table or work backward) you will use.

3) *Carrying out the plan:*

   Perform the necessary computations and describe the steps that you take.

4) *Evaluate solutions:*

   Check if there might be other solutions or other strategies which will yield the same solution.

Student teachers must indicate all questions, attempts, frustrations or any restrictions they may have placed on a problem. Within the context of solving a given set of problems, probing questions are posed such as: Are all the given data relevant to the solution? Do any assumptions have to be made? Are there different ways interpreting the given information or conditions? As the questions are posed, students reach a good understanding of each problem.

The most important step is to encourage students to "generate an extension of the given problem" or "posing a related problem" as is suggested by Gonzales (1994). She suggested a fifth step which is:

5) *Posing a related problem:*

   Use the given problem and modify it to obtain a variation of the given problem. A student poses a related problem by changing the values of the given data and by changing the context of the original problem, that doesn't mean he has to modify or change the solving strategy used in the original problem.

Student teachers may use any of the following techniques in writing new related problems:

Change the values of the given data,

Change the context, and

Change the number of conditions that relate to the problem.

Third: The Group C students have studied "problem solving activities" during the period September to October 2000 (5-weeks) while Group E had studied "problem solving-posing activities" during the period September to October 2000 (7-weeks), the extra two weeks was because of the techniques used in problem posing during and after solving the problems given.

Fourth: An achievement test on "Mathematical problem solving-posing test" was presented to student teachers in Groups C and E in the same period on October 29, 2000 as a post-test.

RESULTS

To determine the effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance, the t-test was used as a measure of comparison between the mean scores on the mathematical problem (solving-posing) test for both the experimental group (E) and the control group (C). The SPSS V10.0 had been used and the results are given in Table1.

Table 1

Means, Standard Deviations and the Value of t Scores in Mathematical Problem (Solving-Posing) Test

Variable	Experimental (n=25)		Control (n=25)		Mean Difference	t-value	P<
	M	SD	M	SD			
Problem solving	4.16	.75	3.64	1	.52	2.09	0.01
Problem posing	2.32	.69	1.76	.66	.56	2.93	0.01
Problem (solving-posing)	8.16	1.43	6.72	1.62	1.44	3.33	0.01

Maximum score in "problem-solving part" of the test = 5 marks
Maximum score in "Problem-posing part" of the test = 3 marks
Maximum score in "Problem (solving-posing) test" = 10 marks

It is evident from Table 1 that the level of problem solving performance has significantly improved for student teachers of the experimental group compared to the performance of other student teachers of the control group in problem solving, with means of 4.16 and 3.64 respectively. The resulting t value of 2.09 is significant at $p<0.01$. This improvement in the level of problem solving performance for the experimental group is consistent with the significant increase in problem posing performance for student teachers in the control group, with means of 2.32 and 1.76 respectively. The resulting t value of 2.93 is significant at $p<0.01$. Student teachers of the experimental group were able to exhibit a significantly higher level of problem-solving performance as compared to that exhibited with student-teachers of control group.

Consistent with the previous result, student teachers of the experimental group were able to exhibit a significantly higher level of problem-posing performance as compared to that exhibited with student-teachers of control group.

Overall, the level of problem (solving-posing) performance has significantly improved for student teachers of the experimental group comparing with performance of student teachers of control group in the test of problem (solving-posing) with means of 8.16 and 6.72 respectively. The resulting t value of 3.33 is significant at $p<0.01$.

DISCUSSION

This study aimed at examining the effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance.

The study sought to develop Polya's four-steps method to include more questions in "make a plan" and "carrying out the plan" or make an extension to the original problem to enhance student teachers' performance in problem solving. Participants of experimental group have studied problem solving using Polya's four-steps as it is developed by problem posing strategies.

Results from this study supported the hypothesis and a significant improvement in student teachers problem solving performance was observed, as well as there is a significant improvement in student teachers' problem posing performance in the achievement test.

An alternative explanation for the present findings is that student-teachers have opportunities to discuss each step in their solving a presented problem with more emphasis on the uses of problem posing strategies used to develop new problem, that support the same findings (Gonzales, 1994; Leung, 1993).

More researches need to be performed on the relationship between problem solving performance and problem posing abilities for students in all stages.

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